6D SCFTs and Integrable Spin Chains

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based on 2007.07262 w/ F. Baume and J.J. Herderman

String Pheno Seminar Series

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Motivation
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(Why am I talking about 6D SCFTs at a string phenom seminar?)
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QFT
Motivation

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QFT + symmetries
Motivation

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QFT + symmetries

Supersymmetry + conformal symmetry

⇒ questions become tractable
Motivation

What kind of questions?

- correlation functions $\langle O_1(x_1) \ldots O_n(x_n) \rangle = ?$

- in CFT correlation functions of local operators are controlled by the scaling dimensions of operators part of the "CFT data"
Motivation

**Why 6D?**

A vast number of 6D $\mathcal{N}=(1,0)$ SCFTs have geometric realizations from string theory. [Heckman, Morrison, Rudelius, Vafa]

- Many theories and lots of information, but unclear how to read off operator dimensions from this construction.
Contents

1) Review of 6D SCFTs from Calabi-Yau geometry
   - what is known about operator dimensions?

2) Rank R Conformal Matter and an Integrable Sector
   - what operator dimensions can we compute?

3) Conclusions
In [Heckman, Morrison, Rudelius, Vafa] it was shown that 6D SCFTs from F-theory take the form of linear quivers, with conformal matter as the links between nodes [del Zotto, Heckman, Tomasiello, Vafa] with some non-linear decorations at the ends.
Since conformal matter behaves as a building block

what do we know the spectrum?

$(\text{SU}(N), \text{SU}(N))$ conformal matter
→ bifundamental hypermultiplet

contains scalar $\phi^a = (X, Y^+)$

$(\text{SO}(k), \text{SO}(k))$ → for $k > 4$ (two half-hypons in $\text{SO}(k) \times \text{Sp}(k-4)$

contains scalar $\psi^i = (\phi^a \otimes \phi^a) = (X^{(i)}, X^{(o)}, X^{(i)})$

$s = \frac{1}{2}$

$s = 1$
Similarly, an analysis of the other strongly coupled conformal matter reveals bifundamental scalar fields

\[ \mathcal{Q}^S = (X^{(S)}, X^{(S-1)}, \ldots, X^{(-S)}) \]

transforming in the spin $S$ rep of SU(2)$_R$.

We find

\[
\begin{array}{c|ccccc}
G & SU(N) & SO(N) & E_6 & E_7 & E_8 \\
\hline
S & 1/2 & 1 & 3/2 & 2 & 3 \\
\end{array}
\]
At leading order triple of D-terms related to quadratic terms in the Xs.

\[
D_{i,a}^R = \frac{1}{s} \times \left( \text{Tr}_{i+1}(X_i^{\dagger(m_i)} S_R^{(m_i,n_j)} T_{i,a} X_i^{(n_j)}) - \text{Tr}_{i-1}(X_{i-1}^{(m_i)} S_R^{(m_i,n_j)} T_{i,a} X_{i-1}^{\dagger(n_j)}) \right) + \ldots
\]

for \( s = \frac{1}{2} \) this is just the minimal coupling between 6D hyper and vector.

for \( s > \frac{1}{2} \) we expect higher order correction (NS-brane fractionation).
An Integrable Sector
Rank $k$ $(G,G)$ conformal matter has generalised quiver

$G_0 \rightarrow G_1 \rightarrow G_2 \rightarrow \ldots \rightarrow G_{k-2} \rightarrow G_{k-1} \rightarrow G_k$

\[ \text{= minimal } (G,G) \text{ conformal matter} \]
As we have seen:

at each — there are bifundamental modes

\[ X_i^{(m_i)} \quad -s \leq m_i \leq s \]

where

<table>
<thead>
<tr>
<th>G</th>
<th>SU(n)</th>
<th>SO(n)</th>
<th>E_6</th>
<th>E_7</th>
<th>E_8</th>
</tr>
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<tbody>
<tr>
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<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>
Construct operators

\[ O = O^{(m_1, \ldots, m_K)} = X_1^{(m_1)} X_2^{(m_2)} \ldots X_K^{(m_K)} \]

bifundamental of flavor \( G_0 \times G_K \)

\[ m_i = S : \quad O_{\text{pure}} = X_1^{(s)} \ldots X_K^{(s)} \]

 scaler of \( D \)-type superconformal multiplet

\[ \Rightarrow \frac{1}{2} - \text{BPS} \Rightarrow \text{no anomalous dimension} = \Delta(O_{\text{pure}}) = 4S(K+1). \]
$R$-charge of $O_{\text{pure}}$:

$$R[O_{\text{pure}}] = (K + 1)S$$

Consider $O$ with "small" number of "impurities"

$$X^{(m_i)} \quad \text{for} \quad m_i \neq s.$$ 

$R[O]$ large when $K \gg 1$.

Treat $1/R$ as expansion parameter.

Long quivers.
Let's start with \((A_n, A_n)\) conformal matter then

\[ \Rightarrow \text{bifundamental hypermultiplet} \]

\[ \Rightarrow \text{we can be very explicit!} \]

For simplicity, focus on single impurity case

\[ \Theta_i = X_0 \cdots X_{i-1} Y_i^+ X_{i+1} \cdots X_k \]

If gauge coupling is switched off, the \(\Delta_i = 2(k+1)\)
If gauge coupling is on \( \Theta_i \) will mix with \( \Theta_j \)

\[
\langle \Theta_i^+(x) \Theta_j(0) \rangle = \frac{1}{1 \times 1^{2\Delta_i}} (\delta_{ij} - R_{ij} \log(1x^2 \lambda^2) + ...) ^{\text{Anomalous dimension matrix}}
\]

Using triplet of D-terms we find, at one loop,

\[
\langle \Theta_i^+(x) \Theta_{i-i}(0) \rangle = \frac{1}{1 \times 1^{2\Delta_0}} \left( 1 + \frac{2g_i^2 \tilde{C}_i}{(4\pi)^2} \int \frac{d^2 \rho}{1 \times 2} \frac{1}{|x-2\Gamma|} \frac{4\Delta_{1\Gamma}}{4\Delta_{1\Gamma}} \right)
\]
Evaluating the integral we find

\[ \langle 0^+ \chi \rangle_{\chi} = \frac{1}{16 \pi^3} \left( 1 + \frac{g_i^2 \tilde{C}}{16 \pi^3} \log(1x^2 \lambda^2) + \ldots \right) \]

Working out all correlators we find

\[ \chi_{ii} = \frac{1}{16 \pi^3} \left( g_i^2 \tilde{C} + g_{i+1}^2 \tilde{C}_{i+1} \right) \]

\[ \chi_{ii+1} = \chi_{i+1,i} = -\frac{g_{i+1}^2 \tilde{C}_{i+1}}{16 \pi^3} \]
We can write this as 1D lattice Laplacian

\[ \nabla = \lambda_A \begin{bmatrix} 1 & -1 \\ -1 & 2 & -1 \\ & -1 & 2 & -1 \\ & & -1 & 2 & -1 \\ & & & -1 & 2 & -1 \\ & & & & -1 & 2 & -1 \end{bmatrix} \]

with \( \lambda_A = \frac{g^2 \tilde{C}_A}{16 \pi^3} \).
Operator mixing by such a lattice Laplacian can be written as a 1D spin chain with $s = \pm \frac{1}{2}$ on each of $k+1$ sites and Hamiltonian

$$H = -\lambda_A \sum_i (2\vec{S}_i \cdot \vec{S}_{i+1} - \frac{1}{2})$$

This is exactly the Hamiltonian for the integrable Heisenberg spin chain $\Rightarrow$ we have found integrability!
Spin Chain State

\[ | \uparrow \ldots \uparrow \uparrow \uparrow \uparrow \ldots \uparrow \rangle \]

\[ X_0 \ldots X_{i-1} X_i X_{i+1} \ldots X_N \]

\[ | \uparrow \ldots \downarrow \uparrow \uparrow \uparrow \ldots \uparrow \rangle \]

\[ X_0 \ldots Y_i^\dagger X_i X_{i+1} \ldots X_N \]

\[ | \uparrow \ldots \uparrow \downarrow \uparrow \uparrow \ldots \uparrow \rangle \]

\[ X_0 \ldots X_{i-1} Y_i^\dagger X_{i+1} \ldots X_N \]

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\[ X_0 \ldots X_{i-1} X_i Y_{i+1}^\dagger \ldots X_N \]
Extending Integrability

The AdS/CFT duals of 6D rank $k$ conformal matter are $\text{AdS}_7 \times S^4/\Gamma$ $\Gamma$ finite $\subset \text{SU}(2)$. Our protected operators are not sensitive to the different $\Gamma$ in the dual → reasonable to expect integrability to persist.
Assumption: operator mixing in this sector is controlled by the

1D integrable open Heisenberg XXX spin chain

also for \( s > \frac{1}{2} \).

dilatation operator \( \leftarrow \) spin chain Hamiltonian

integrability fixes this Hamiltonian uniquely! [Babujian]
\[ H_s = -\lambda_s \sum_{i=0}^{k-1} Q_{2s}(\vec{S}_i \cdot \vec{S}_{i+1}) \]

where

\[ \vec{S}_i = \text{spin } s \text{ operator} \]

\[ Q_{2s} = \text{degree } 2s \text{ polynomial} \]

\[
Q_{2s}(x) = -2 \sum_{l=0}^{2s} \sum_{k=l+1}^{2s} \frac{1}{k} \prod_{j=0}^{2s} \frac{x - x_j}{x_l - x_j}, \quad \text{with } x_l = \frac{1}{2}l(l+1) - s(s+1)
\]
\[ Q_{A_k}(x) = -\frac{1}{2} + 2x \]
\[ Q_{D_k}(x) = \frac{1}{2} x - \frac{1}{2} x^2 \]
\[ Q_{E_6}(x) = -\frac{3}{4} - \frac{1}{8} x + \frac{1}{27} x^2 + \frac{2}{27} x^3 \]
\[ Q_{E_7}(x) = -\frac{1}{2} + \frac{13}{24} x + \frac{43}{432} x^2 - \frac{5}{216} x^3 - \frac{1}{144} x^4 \]
\[ Q_{E_8}(x) = -\frac{148}{125} - \frac{1687}{9000} x + \frac{1297}{18000} x^2 + \frac{593}{20250} x^3 + \frac{79}{97200} x^4 - \frac{77}{243000} x^5 - \frac{1}{48600} x^6 \]

For \( s > \frac{1}{2} \) there are higher order spin-spin interaction terms.

\[ \rightarrow \text{consistent w/ "fractionation" of M5-branes for (O,O) and (E,E) conformal matter.} \]
Bethe Ansatz

Consider operator $O$ with $I$ impurities:

\[ e^{i\rho_i} = \frac{\mu_j + is}{\mu_j - is} \]

\[ \text{momenta} \]
\[ \text{rapidity} \]

Bethe ansatz equations:

\[ \left(\frac{\mu_j + is}{\mu_j - is}\right)^2 = \prod_{l \neq j} \frac{(\mu_j - \mu_l + i)(\mu_j + \mu_l + i)}{(\mu_j - \mu_l - i)(\mu_j + \mu_l - i)} \]

Decoupling:

\[ \sum_i \rho_i = 0 \implies \prod_{j=1}^I \frac{\mu_j + is}{\mu_j - is} = 1 \]

Energy/anomalous dimension}

\[ \Delta - \Delta_0 = E_B = \lambda_0 \sum_{j=1}^I \left( \frac{i}{\mu_j + is} - \frac{i}{\mu_j - is} \right) \]
Example: $G = SU(N)$, $I = 2$

Two momenta $p_1, p_2$

$\rightarrow$ decoupling equation $\Rightarrow p_1 = -p_2 = \mu$

BAE becomes:

$\left(\frac{\mu + i/2}{\mu - i/2}\right)^{2(k+1)} = \frac{2\mu + i}{2\mu - i} = \frac{\mu + i/2}{\mu - i/2}$

$\Rightarrow p_1 = -p_2 = \frac{2\pi m}{2k + 1}$ for $m = 0, \ldots, k-1$

Anomalous dimensions: $\Delta - \Delta_0 = \lambda \frac{1}{2} \times 8 \sin^2 \left( \frac{\pi m}{2k + 1} \right)$. 
Conclusions

1) Uncovered one-loop integrability in certain large R-charge sectors of 6D SCFTs.
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1) Uncovered one-loop integrability in certain large R-charge sectors of 6D SCFTs.  

   → as a byproduct: we determined the anomalous dimension of an interesting class of "nearly protected" operators.
Future

- how far does this integrable structure extend?
- higher loops?
- different operator sectors/impurities?
- connections with AdS/CFT?
- ...

Work to appear w/ Baume and Heckman
Thank you