Control Issues of KKLT

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Outline

Introduction

The singular-bulk problem

Escape routes

Conclusions
Introduction
dS vacua in string theory?

Long debated question: Is dS possible in string theory?

- Plausible and much studied scenarios such as KKLT, LVS
  But not fully explicit

- Many no-go theorems in particular corners of string theory

No-dS conspiracy?

e.g. (refined) dS conjecture

Can this be true?
→ Crucial to construct explicit models realizing the scenarios
   or identify potential problems

Today: focus on earliest and most studied proposal, the KKLT scenario
Proposal: meta-stable dS vacua in 3 steps:

- IIB flux vacua with strongly warped throat modelled locally by Klebanov-Strassler solution
  
  Fluxes $K, M$ carry D3 charge $N = KM$
  localized at the tip

- Kähler modulus $T$ stabilized by non-perturbative effects (E3 instanton or gaugino condensate on $N_c$ D7 branes)

  SUSY AdS vacua with vacuum energy density

  \[ V_{\text{AdS}} \sim -e^{-\text{Re}(T)/N_c} \]

  (up to non-exponential effects)

In the following: set $N_c = 1$ (comments on $N_c \neq 1$ later)
Uplift to dS by placing anti-D3 brane in the throat energy density redshifted due to strong warping

\[ V_{\text{uplift}} \sim e^{-K/g_sM} \]

Meta-stable if uplift energy is not too large:

\[ V_{\text{uplift}} \sim |V_{\text{AdS}}| \quad \leftrightarrow \quad e^{-K/g_sM} \sim e^{-\text{Re}(T)} \]

\[ \text{Re}(T) \sim \frac{N}{g_sM^2} \]

\( g_sM \gtrsim 1 \) (small curvature at KS tip), \( M > 12 \) (meta-stability)

\( g_sM^2 > (6.8)^2 \) (conifold)

→ Treat \( g_sM^2 \gg 1 \) as large parameter

Klebanov, Strassler 00
Kachru, Pearson, Verlinde 01

Bena, Dudas, Graña, Lüst 18
Blumenhagen, Kläwer, Schlechter 19
Bena, Buchel, Lüst 19; Dudas, Lüst 19
Randall 19
Observation:
For $g_s M^2 \gg 1$, strongly warped throat *does not “fit”* into weakly warped CY bulk
Possible threat of large singularities

But is this really a problem?
A priori, strong warping can be fine with supergravity approximation

→ Need to study warped geometry for $g_s M^2 \gg 1$
The singular-bulk problem
Constraint on the warp factor

IIB on (conformally) CY orientifold $X$ with (string-frame) metric

\[ ds_{10}^2 = h(y)^{-1/2} \eta_{\mu\nu} dx^\mu dx^\nu + h(y)^{1/2} \tilde{g}_{mn} dy^m dy^n \]

warp factor

Ricci-flat, \( \tilde{V}_X \equiv \int_X d^6 y \sqrt{\tilde{g}} = 1 \)

Kähler modulus $T$ is defined in terms of (Einstein-frame) 4-cycle volume wrapped by E3:

\[
\text{Re}(T) \sim S_{E3} \sim \frac{N}{g_s M^2} \quad \Leftrightarrow \quad \frac{1}{g_s} \int_\Sigma d^4 \xi \sqrt{\tilde{g}} h \sim \frac{N}{g_s M^2} \quad (2\pi \sqrt{\alpha'} = 1)
\]

with \( \int_\Sigma d^4 \xi \sqrt{\tilde{g}} \gtrsim k_{111}^{1/3} \tilde{V}_X^{2/3} \sim O(1) \) in our normalization

\[ \langle h \rangle_\Sigma \sim \frac{N}{M^2} \]

\[ \langle h \rangle_\Sigma \equiv \frac{\int_\Sigma d^4 \xi \sqrt{\tilde{g}} h}{\int_\Sigma d^4 \xi \sqrt{\tilde{g}}} \]

\[ \rightarrow \text{warp-factor average over } \Sigma: \]
Warp-factor variation

\[ \langle h \rangle_\Sigma \sim \frac{N}{M^2} \] implies a neighborhood on \( \Sigma \) where

\[ h \lesssim \frac{N}{M^2} \]

Warp factor satisfies **Poisson equation**:

\[ \tilde{\nabla}^2 h = -g_s \tilde{\rho}_{D3} \]

D3-charge density

**Variation** of the warp factor due to D3 charge \( N \) at the KS tip:

\[ |\nabla h| \sim g_s N \quad \text{(at } O(1) \text{ distance in } \tilde{g}) \]

with \( |\nabla h| \equiv \sqrt{(\partial_m h)(\partial_n h)\tilde{g}^{mn}} \)

\[ \rightarrow \text{neighborhood on } \Sigma \text{ with } \frac{|\nabla h|}{h} \gtrsim g_s M^2 \gg 1 \]
Singularity

Use $\frac{\partial h}{h} \gtrsim g_s M^2$ with $h(y_0 + \delta y) \approx h(y_0) + \partial_m h(y_0) \delta y^m$

Recall $h = g_s N \times "O(1) function"

$\Rightarrow$ singularity $h \leq 0$ at $|\delta y| \lesssim 1/g_s M^2 \ll 1$

Recap:
- Step 1 of the KKLT proposal requires a flux compactification with a volume modulus $\text{Re}(T) \sim \frac{N}{g_s M^2}$ and a conifold region hosting a D3 charge $N$
- $\text{Re}(T)$ is too small (relative to $N$) to ensure small curvature; instead, the D3 charge creates singularities in the bulk
Size of the singularity

How large is the singular region?

Variation of $h$ much larger than its average $\langle h \rangle_\Sigma \sim \frac{N}{M^2}$

$\rightarrow h < 0$ on $O(1)$ fraction of E3 volume (in $\tilde{g}$)

Generically, it will then also spread over an $O(1)$ distance into the transverse space

Complementary argument: coarse-grained warp factor (see paper)
Escape routes
Escape routes?

- **Special geometries** avoiding our parametric estimates e.g. screen KS charge by special O-plane arrangement

- **Large** $N_c$ helps:

  \[
  V_{\text{AdS}} \sim -e^{-\text{Re}(T)/N_c} \iff \frac{|\partial h|}{h} \gtrsim \frac{g_s M^2}{N_c}
  \]

  But: D7 tadpole constraints bound $N_c < O(10) \, h^{1,1}$

- **Variants of KKLT with** $h^{1,1} \neq 1$

  All models suffer from a singular-bulk problem

  Possible exception:

  Parametrically large $h^{1,1} \gtrsim (g_s M^2)^{3/5} \gg 1$

  and $N_c \sim O(h^{1,1})$ D7 stack on most 4-cycles (so $O((h^{1,1})^2)$ D7 branes in total)

  D7 tadpole?
Conclusions
Conclusions

- Flux compactifications admitting a KKLT-like dS uplift generically have large singularities that extend over an $O(1)$ fraction of the Calabi-Yau

- The singularities arise because the charge $N$ in the KS throat leads to a too large variation of the warp factor in the bulk

- Difficult to escape the conclusion

- LVS appears to avoid the problem. Could there be other hidden problems preventing explicit models?
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Thank you!
Size of the singularity

At large $N$, we can consider a coarse-grained warp factor $h_c(y)$

$$ \text{with} \quad d_{O3} \ll d \ll 1 \quad (\text{in } \tilde{g})$$

Coarse-grained D3-charge distribution:
Positively-charged lump of diameter $d$ at the tip of the conifold,
uniform negative charge density

$\rightarrow$ Negative “spikes” due to O-planes averaged away in $h_c$

$h_c$ singular by the same arguments as above

$$\left| \frac{\partial h_c}{h_c} \right| \gg 1$$
Toy model

- Compact space: $S^6$ with polar angle $\phi \in (0, \pi)$
- Point-like source with charge $N$ at $\phi = 0$ ("KS throat")
- Add uniform negative charge density to satisfy Gauss law ("O-planes")

Poisson equation $\tilde{V}^2 h = -g_s \tilde{\rho}_{D3}$ becomes:

$$\pi^3 [\sin^5 \phi \ h(\phi)']' = -g_s N \left( \delta(\phi) - \frac{15}{16} \sin^5 \phi \right)$$

Solution: $h(\phi) = g_s N h_0(\phi) + \text{const.}$ $h_0(\phi) \sim O(1)$

Fix constant by condition on $h$ at “instanton” position $\phi = \phi_{E3}$:

$$h(\phi_{E3}) \sim \frac{N}{M^2}$$