Infinite Distance, Emergent Strings and Quantum Corrections in 4D $\mathcal{N} = 1$


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Outline

• Swampland essentials
• Infinite distance limits in F-theory on $CY_4$
• Uniqueness of emergent string
• Quantum obstructions
• Weak Gravity Conjecture
Swampland Essentials
Swampland Program

Vafa '05
Review: Palti '19

Landscape:
EFT + quantum gravity = ✓

Swampland:
EFT + quantum gravity = ⚡

Swampland conjectures:
landscape vs. swampland
Distance Conjecture

Ooguri, Vafa ‘06

∃! Infinite tower of states with \( m \sim e^{-\alpha \Delta \phi} M_p \)
Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, Vafa ‘06

Extremal BH should decay ⇒ Need particle with $m \lesssim gqM_p$

$F \sim (m/M_p)^2$

$F \sim (gq)^2$
Weak Coupling At Infinite Distance

Heidenreich, Reece, Rudelius ’15, ’16, ’17, ’18
DK, Palti ’16
Lee, Lerche, Weigand ’18, ’19

- Connection: if $g \sim e^{-\gamma \phi}$, sub-lattice WGC predicts objects with $m_n \lesssim q_n e^{-\alpha \phi} M_p$

- Both consequence of approaching $\infty$ distance loci in field space

Tower

Kaluza-Klein

$\begin{align*}
  m_n &\sim n m_0 \\
\end{align*}$

String

$\begin{align*}
  m_n &\sim \sqrt{n} m_0 \\
\end{align*}$
Emergent String Conjecture
Lee, Lerche, Weigand ’19; DK, Lee, Weigand, Wiesner ‘20

Infinite distance limits in the moduli space of a consistent theory of quantum gravity fall into two classes:

A) Decompactification (KK tower dominant)

B) Emergent String (tensionless critical string dominant)
Emergent String Conjecture

This has been checked in many settings:

F-Theory on $CY_3$ Kähler moduli: D3 on $\mathbb{P}^1$  
Lee, Lerche, Weigand '18

Type IIB on K3: D3 on $E_\tau$  
Lee, Lerche, Weigand '19

Type IIB on $CY_3$ hypermultiplet moduli: F1, D1, D3 on $E_\tau$  
Baume, Marchesano, Wiesner '19

IIA / M-Theory on $CY_3$ Kähler moduli: NS5 / M5 on $K3$ or $T^4$  
Lee, Lerche, Weigand '19

**F-Theory on $CY_4$ Kähler moduli: D3 on curve $C_0$**  
DK, Lee, Weigand, Wiesner '20
4D N=1 F-Theory

Infinite Distance Limits
F-Theory Realisation

Lee, Lerche, Weigand ’18, ’19

• F-Theory on elliptic $E_\tau \rightarrow CY_4 \rightarrow B_3 \Rightarrow 4D \, \mathcal{N} = 1$

• Interested in $h^{1,1}(B_3)$ Kähler moduli

$$J' = (v')^\alpha J_\alpha \Rightarrow \mathcal{V}_{B_3} = \frac{1}{3!} \int (J')^3 = \frac{1}{3!} k_{\alpha \beta \gamma} (v')^\alpha (v')^\beta (v')^\gamma$$

• Trivial $\infty$ distance limit: $J' = \mu J$ with $\mu \rightarrow \infty$

$\rightarrow$ decompactification with $M_{KK}/M_s \sim \mu^{-1/2} \rightarrow 0$

• If $J_0^3 = 0$ for some $J_0$ we may have non-trivial finite volume limit

• General limit will be superposition:

$$J' = \mu \left( \lambda J_0 + \sum_i v_i(\lambda) J_i \right) = \mu J$$
F-Theory Realisation

Lee, Lerche, Weigand ’18, ’19

\[ J = \lambda J_0 + \sum_{\mu \in \mathcal{I}_1} v^\mu J_\mu + \sum_{\nu \in \mathcal{I}_2} v^\nu J_\nu + \sum_{r \in \mathcal{I}_3} v^r J_r \]

\[ J_0^2 \cdot J_\mu \neq 0 \quad \forall \mu \in \mathcal{I}_1 \]

\[ J_0^3 = 0 \]

\[ J_0^2 \cdot J_\nu = 0 \quad \forall \nu \in \mathcal{I}_2 \quad \text{and} \quad \exists \nu' \in \mathcal{I}_2: J_0 \cdot J_\nu \cdot J_{\nu'} \neq 0 \]

\[ J_0^2 \cdot J_r = 0 \quad \forall r \in \mathcal{I}_3 \quad \text{and} \quad \forall i \in \mathcal{I}_2 \cup \mathcal{I}_3: J_0 \cdot J_r \cdot J_i = 0 \]
J-class A and J-class B

Lee, Lerche, Weigand '18, '19
Focus: J-class A

\[ J^3_0 = 0 \quad J^2_0 \cdot J_1 \neq 0 \quad J^2_0 \cdot J_3 = J_0 \cdot J^2_3 = J^3_3 = 0 \]

\[ \nu^3 \sim \mathcal{O}(1) \]

\[ \nu^1 \sim \frac{1}{\lambda^2} \]

\[ \mathcal{Y}_{B_3} \sim J^3 \sim \lambda^0 \cdot \frac{J^2_0 \cdot J_1}{\lambda} + \mathcal{O}\left(\frac{1}{\lambda}\right) \]
We have a distinguished curve class $C_0 = J_0 \cdot J_0$ with $
abla C_0 \sim \frac{1}{\lambda^2} \to 0$

Two options: $\hat{K}_{B_3} \cdot C_0 = 2 - 2g \geq 0$

1) $\hat{K}_{B_3} \cdot C_0 = 2 \implies \mathbb{P}^1 \to B_3 \to B_2$

2) $\hat{K}_{B_3} \cdot C_0 = 0 \implies T^2 \to B_3 \to B_2$

Here we focus on the first case

D3-brane wrapped on $C_0$ gives rise to emergent heterotic string
Some Puzzles...

• Multiple emergent strings becoming light at the same rate?

\[ B_3 = \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1 \quad (\text{J-class B}) \]

\[ \lambda \quad \frac{1}{\lambda^{1/2}} \quad \frac{1}{\lambda^{1/2}} \]

• Compute: \( M_{KK}^2 / M_{\text{het}}^2 \sim \lambda \to \infty \)

→ Naively, this limit decouples the KK tower and we are left with an intrinsically 4D N=1 heterotic string
Uniqueness of Emergent String
Uniqueness of Emergent String

DK, Lee, Weigand, Wiesner ‘20

Back to our example: \[ B_3 = \underbrace{\mathbb{P}^1_A}_\lambda \times \underbrace{\mathbb{P}^1_B}_{1/\lambda^{1/2}} \times \underbrace{\mathbb{P}^1_C}_{1/\lambda^{1/2}} \]

- We have: \[ M_{KK}^2 \sim \frac{M_s^2}{\mathcal{V}_{\mathbb{P}^1_A}} \sim \frac{M_s^2}{\lambda} \quad T_{\text{str},A} \sim T_{\text{str},B} \sim M_s^2 \cdot \mathcal{V}_{\mathbb{P}^1_A} \sim \frac{M_s^2}{\lambda^{1/2}} \]

- Clearly the **KK tower dominates** and we never enter the heterotic duality frame.

- We prove that this is the case in general.
Uniqueness of Emergent String

DK, Lee, Weigand, Wiesner ‘20

Sketch of Proof (J-class A):

• Suppose $B_3$ admits another rational or genus-one fibration with generic fiber $\tilde{C}_0$ that shrinks in the limit.

  $\Rightarrow \exists$ nef divisor $\tilde{D} \in \overline{\mathcal{H}(B_3)}$ such that $\tilde{D}^2 = \tilde{n} \tilde{C}_0$ and $\tilde{D}^3 = 0$.

• We can expand $\tilde{D} = p^0 J_0 + \sum_{\mu \in \mathcal{I}_1} p^\mu J_\mu + \sum_{\mu \in \mathcal{I}_3} p^r J_r$

• We show that at least one $p^{\mu_0} > 0$ and $p^0 = p^r = 0$

• Now one can show $\nabla_{\tilde{D}^2} = J' \cdot \tilde{D}^2 \rightarrow 0$ implies $\kappa_{00\mu_0} = 0 \Rightarrow$ contradiction
Quantum Obstructions
Quantum Corrections

Shrinking divisor volumes ⇒ Corrections to the effective action, e.g.

1) Perturbative $\alpha'$-corrections to the Kähler potential

2) Non-perturbative (instanton) corrections to the superpotential

The superpotential 2) can receive corrections from D3-branes wrapping vertical divisors of the fibration of $B_3$. Intuitively, at least for the shrinking $\mathbb{P}^1$ limit, we nevertheless expect that the heterotic superpotential vanishes.

$$W_{np} \sim e^{-S_{het}} \rightarrow 0$$

We argue that the superpotential indeed vanishes in all limits which are not already obstructed by 1).

⇒ study perturbative $\alpha'$-corrections
The $\alpha'$-Obstruction

Higher derivative corrections to M-theory lead to corrections to the 4D F-theory Kähler potential and coordinates

\[ e^{K/2} = \mathcal{V}_{B_3} = \mathcal{V}_{B_3}^0 + \alpha^2 \left( (\tilde{\kappa}_1 + \tilde{\kappa}_2) \mathcal{E} + \tilde{\kappa}_2 \mathcal{T} \right) \]

\[ T_{\alpha} = \frac{K_{\alpha}}{2} + \alpha^2 \left( (\kappa_3 + \kappa_5) \frac{K_{\alpha} \mathcal{E}}{2 \mathcal{V}_{B_3}^0} + \kappa_5 \frac{K_{\alpha} \mathcal{T}}{2 \mathcal{V}_{B_3}^0} + \kappa_4 \mathcal{E}_{\alpha} \log(\mathcal{V}_{B_3}) + \kappa_6 \mathcal{T}_{\alpha} + \kappa_7 \mathcal{E}_{\alpha} \right) \]

\[ \mathcal{E}_{\alpha} = \int_{Y_4} c_3(Y_4) \wedge \pi^*(J_{\alpha}) , \quad \mathcal{E} = \nu^\alpha \mathcal{E}_{\alpha} \]

\[ \mathcal{T}_{\alpha} = -18(1 + \alpha_2) \frac{1}{T_{\alpha}} \int_{D_{\alpha}} c_1(B_3) \wedge J \int_{D_{\alpha}} J_{\alpha} \wedge J , \quad \mathcal{T} = \nu^\alpha \mathcal{T}_{\alpha} \]

*smooth Weierstrass model
The $\alpha'$-Obstruction

\[ e^{\kappa/2} = \mathcal{V}_{B_3}^\prime = \mathcal{V}_{B_3}^0 + \alpha^2 \left( (\tilde{\kappa}_1 + \tilde{\kappa}_2) \mathcal{E} + \tilde{\kappa}_2 \mathcal{T} \right) \]

- Perturbative control: $\frac{\mathcal{E}}{\mathcal{V}_{B_3}^0} \ll 1$ and $\frac{\mathcal{T}}{\mathcal{V}_{B_3}^0} \ll 1$

- In our limit $J' = \mu \left( \lambda J_0 + \frac{a}{\lambda^2} J_1 + b J_3 \right)$ ( $\mu = 1$ is the finite volume limit )

\[ \Rightarrow \quad \frac{\mathcal{E}}{\mathcal{V}_{B_3}^0} \sim \frac{\mu \lambda \mathcal{E}_0 + \ldots}{\mu^3} \sim \frac{\lambda}{\mu^2} \mathcal{E}_0 \]

- The finite volume limit is not under control! $\mu \sim \lambda^{1/2}$ is marginally OK

- For this limit, the heterotic string and KK scale coincide parametrically

\[ \frac{M_{KK}}{M_{het}} \to \text{const.} \]
$\Delta_0 \simeq \Delta_1$
decompactification

$M_{\text{KK}}^2 \ll T_{\text{str}}$

$\mu^2 \sim \lambda$
unique emergent string

$T_{\text{str}} \sim M_{\text{KK}}^2$

$\Delta_0 \ll \Delta_1$
obstructured

$T_{\text{str}} \ll M_{\text{KK}}^2$
Weak Gravity Conjecture
Emergent String and WGC

Lee, Lerche, Weigand ’18, ’19

• Any divisor $S$ with $S \cdot C_0 \equiv 2m \neq 0$ grows in the limit as $\mathcal{V}_S \sim \mu^2 \lambda^2$

• If $S$ is wrapped by a 7-brane: weak coupling limit for gauge theory

• In order to prove the WGC we need two ingredients:

\[
\frac{\mathcal{V}_{C_0} \cdot \mathcal{V}_S}{M_{\text{het}}^2 \frac{1}{g_{YM}^2}} = 2m \mathcal{V}_{B_3} M_p^2
\]

classical geometry of the limit

$\exists$ tower of states with $q_k^2 \geq 4mn_k$

elliptic genus

$M_k \geq g_{YM} q_k M_{\text{pl}}$
WGC States from the Elliptic Genus

Lee, Lerche, Weigand ’18, ’19; DK, Lee, Weigand, Wiesner ‘20

• A tower of WGC states has already been identified in Lee, Lerche, Weigand ’18, ’19.

• The relevant states can be read off from the elliptic genus.

• In 6D the elliptic genus is a meromorphic Jacobi-form. This property was used in Lee, Lerche, Weigand ’18 to prove sub-lattice WGC for F-theory on $CY_3$.

• In 4D the elliptic genus is generally not a Jacobi-form, so the result does not carry over directly. As a result, Lee, Lerche, Weigand ’18 were only able to identify a tower as opposed to a sub-lattice of states generically.

• Building on recent results Lee, Lerche, Weigand ’20 on the decomposition of the 4D elliptic genus in terms of Jacobi-form building blocks we are now able to show that the sub-lattice WGC holds for generic $U(1)$ fluxes! In the non-generic case, the sub-lattice may be shifted.
$\alpha'$-Corrections and the WGC

- Another subtlety in 4D comes from $\alpha'$-corrections to the relation $\mathcal{V}_c \cdot \mathcal{V}_s = 2m \mathcal{V}_B$.

- We find $\frac{\mathcal{V}_s \cdot \mathcal{V}_c}{2m \mathcal{V}_B} = 1 + \Delta_0 + \Delta_1 + \ldots$,

where $\Delta_0$ represents corrections from the classical geometry away from the limit and $\Delta_1$ are the leading $\alpha'$-corrections. They can be mapped to heterotic threshold corrections.

- Before including $\alpha'$-corrections the WGC tower satisfies the repulsive force condition

$$|F_{\text{Coulomb}}| \geq |F_{\text{Grav}}| + |F_{\text{Yuk}}| \quad \Rightarrow \quad g_{YM}^2 q_k^2 \geq \frac{M_k^2}{M_{pl}^2} \left( \frac{d-3}{d-2} + \frac{1}{4} \frac{M_{pl}^4}{M_k^4} g^{rs} \partial_r \left( \frac{M_k^2}{M_{pl}^2} \right) \partial_s \left( \frac{M_k^2}{M_{pl}^2} \right) \right)$$

- If we demand that this still holds after including the $\alpha'$-corrections, this implies that the masses $M_k$ are also corrected.

As a result, we conclude that the WGC relation is modified:

$$\frac{g_{YM}^2 q_k^2}{M_k^2} \geq \frac{1}{M_{pl}^2} \left( 1 - \frac{1}{2} (\Delta_0 + \Delta_1) \right)$$
Summary

• Infinite distance limits in the Kähler moduli space of F-Theory on elliptic $CY_4$ can be classified by the dominant tower of states that becomes light:

1. Excitations of unique heterotic string

2. Excitations of unique type II string

3. Kaluza-Klein tower

• In a regime of controlled $\alpha'$-corrections, the KK tower can never be decoupled.

• Generically, we can show that the excitations of the heterotic string furnish a sub-lattice WGC tower. We analyse $\alpha'$-corrections to the WGC relation.
Thank You!