The Calabi-Yau Landscape: Beyond the Lampposts

Mehmet Demirtas
Cornell University

String Pheno Series, 2020

Based on works with (various subsets of):
Manki Kim, Cody Long, Liam McAllister, Jakob Moritz,
Mike Stillman, Andres Rios Tascon
What is possible in quantum gravity?

- de-Sitter solutions?
- Super-Planckian field ranges?

- Quintessence?
- Global symmetries?
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• Ultralight axions?
• Light dark sectors?
• Exponential hierarchies?
• Light moduli?
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Can answer for: Weakly coupled compactifications of superstring theories.
To get started: Compactifications on *simple* Calabi-Yau (CY) manifolds with *small* Hodge numbers.
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However: this is an *exponentially small* fraction of the String Landscape.

- Number of (known) topologically inequivalent CY manifolds increases exponentially with $h^{1,1}$.
  
  [MD, McAllister, Rios Tascon, hep-th/2008.01730]

- Number of flux vacua in type IIB (F-Theory) compactifications increases exponentially with $h^{2,1} (h^{3,1})$.
  
  [Denef, Douglas, hep-th/0404116]
  [Denef, Douglas, hep-th/0411183]
  [Taylor, Wang, hep-th/1511.03209]
We can now construct CY threefolds with largest known Hodge numbers and compute relevant topological data.
Outline

I. CY$_3$’s from Triangulations

II. Holomorphic Cycles
   Application: Ultralight Axions

III. 3-cycles
   Application: Towards KKLT
A Quick Review

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• Largest known set of CY threefolds: hypersurfaces in toric varieties.

[Batyrev, alg-geom/9310003]
[Kreuzer, Skarke, hep-th/0002240]
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The construction:  

1. Take a 4D reflexive lattice polytope
   Reflexive: the only interior point of the polytope (and its dual) is the origin.
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This triangulation defines a fan, which describes a toric variety $V$ that has a CY hypersurface $X$.

[Batyrev, alg-geom/9310003]
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How many CY\(_3\) hypersurfaces are there?
- Not known.
- We recently proved an upper bound of \(10^{428}\). [MD, McAllister, Rios Tascon, hep-th/2008.01730]
Holomorphic Cycles

**Notation:**

- $V$: 4D Ambient Variety, $V \supset X$: Calabi-Yau threefold hypersurface
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• \( \mathcal{M}(X) \), \( \mathcal{K}(X) \) are dual cones:

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\mathcal{K}(X) = \left\{ J \in H^{1,1}(X, \mathbb{R}) \left| \int_C J \geq 0 \forall C \in \mathcal{M}(X) \right. \right\}
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Holomorphic Cycles

• Volumes of 2-cycles $C$, 4-cycles $D$, and $X$ itself

$$\text{Vol}(C) = \int_C J \quad \text{Vol}(D) = \frac{1}{2} \int_D J \wedge J \quad \text{Vol}(X) = \frac{1}{6} \int_X J \wedge J \wedge J$$

are determined by the Kähler form $J$ and the intersection numbers:

$$\kappa_{ijk} = \# D_i \cap D_j \cap D_k \quad \text{where } \{D_i\} \text{ span } H_4(X, \mathbb{Z}).$$
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- \textit{Stretched} Kähler cone:

$$\tilde{\mathcal{K}}(X) := \left\{ J \in H^{1,1}(X, \mathbb{R}) \left| \int_C J \geq 1 \forall C \in \mathcal{M}(X) \right. \right\} \quad (2\pi)^2 \alpha' \equiv \ell_s^2 \to 1$$
Kähler cone generator

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estimate for the convergence of the worldsheet instanton expansion and the control of the $\alpha'$ expansion.

[Candelas, De La Ossa, Green, Parkes, '90]
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[Braun, Walliser, hep-th/1106.4529]  
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• $h^{1,1} = \mathcal{O}(100)$: Only recently.
  - [MD, Long, McAllister, Stillman, hep-th/1808.01282]
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### Obtain one triangulation

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A software package for constructing CY hypersurfaces in toric varieties.

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Can compute:

- Lattice points on the polytope
- The dual polytope
- Faces, dual faces of the polytope
- GLSM charge matrix
- Stanley-Reisner ideal
- Second Chern class
- Mori cone of the ambient variety
- Stretched Kähler cone
- Volumes of cycles
- … many more!

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Aside: Some of these quantities can be predicted using Machine Learning.

- Achieved using a deep neural net. High precision even at $h^{1,1} = 491$.
- 50µs per prediction. A further speed-up of a factor of $\sim 10,000$. [MD, McAllister, Rios Tascon, hep-th/2008.01730]
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1. Construct Ensembles of Geometries
2. Detect Patterns
3. Study Consequences
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[MD, Long, McAllister, Stillman, hep-th/1808.01282]
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**Pattern:** At large $h^{1,1}$, **Kähler cones are narrow.**

- Stretched Kähler cone is far away from the origin.
- **Volumes** of effective 2-cycles, 4-cycles and the CY itself are **large.**

[MD, Long, McAllister, Stillman, hep-th/1808.01282]
Consequences:
Consider type IIB compactified on an O3/O7 orientifold of X.
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$C_4$ axions: $\theta_i := \int_{D_i} C_4$,

get a potential from non-perturbative objects (ED3s, D7 branes) wrapping 4-cycles,

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Further Consequences:
• Hierarchies in 4-cycle volumes $\rightarrow$ Realizing KKLT is hard.
• Large 4-cycles $\rightarrow$ Rich 7-brane dark sector. [Cvetic, Halverson, Lin, Long, hep-th/2004.00630]
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get a potential from non-perturbative objects (ED3s, D7 branes) wrapping 4-cycles,

\[ V(\theta) \sim e^{-2\pi \text{Vol}(D)} \]

Large 4-cycles \( \rightarrow \) Suppressed potential \( \rightarrow \) Ultralight axions!
\( \rightarrow \) Black hole superradiance (See Viraf’s talk!)  

Further Consequences:

• Hierarchies in 4-cycle volumes \( \rightarrow \) Realizing KKLT is hard.
• Large 4-cycles \( \rightarrow \) Rich 7-brane dark sector.  
• Implications for fitting warped throats in compactifications

[MD, Long, McAllister, Stillman, hep-th/1808.01282]
[MD, Long, Marsh, McAllister, Mehta, Stott, work in progress]
[Carta, Moritz, Westphal, hep-th/1902.01412]
Periods of 3-cycles

To study flux compactifications, we need to compute the periods of 3-cycles.
• Pick a basis of $H_3(X, \mathbb{Z}), \{A^i, B_j\}$ such that
  \[
  A^i \cap B_j = -B_j \cap A^i = \delta^i_j \quad \text{and} \quad A^i \cap A^j = B_i \cap B_j = 0
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• Periods:

$$\vec{\Pi}(\vec{u}) = \left( \int_{A^i} \Omega(\vec{u}) \right. \left. \int_{B_i} \Omega(\vec{u}) \right)$$

where $\Omega(\vec{u})$ is the holomorphic 3-form and $\vec{u}$ are the complex structure moduli.
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- Can be written in terms of a prepotential $\mathcal{F}$:

$$\vec{\Pi}(\vec{u}) = \begin{pmatrix} 2\mathcal{F} - u^a \partial_a \mathcal{F} \\ \partial_a \mathcal{F} \\ 1 \\ u^a \end{pmatrix}$$
Periods of 3-cycles

- Around a large complex structure point,

\[
\mathcal{F}(\vec{u}) = \mathcal{F}_{\text{poly}}(\vec{u}) + \mathcal{F}_{\text{exp}}(\vec{u})
\]

\[
\mathcal{F}_{\text{poly}}(\vec{u}) = -\frac{1}{3!} \tilde{\kappa}_{abc} u^a u^b u^c + \frac{1}{2} a_{ab} u^a u^b + b_a u^b + \frac{\zeta(3) \chi(\tilde{X})}{2(2\pi i)^3}, \quad \tilde{X}: \text{mirror of } X
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• To compute the periods we need to identify an \textbf{integral basis of 3-cycles.}
Periods of 3-cycles

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- To compute the periods we need to identify an integral basis of 3-cycles.

Mirror Symmetry:

\[
\begin{align*}
\text{Integral} & \quad \text{3-cycles in } X \\
\text{Integral} & \quad \text{2n-cycles in } \tilde{X}
\end{align*}
\]

- Integral 3-cycles in X
- Integral 2n-cycles in \(\tilde{X}\)
Periods of 3-cycles

• Around a large complex structure point,
  \[ F(\vec{u}) = F_{\text{poly}}(\vec{u}) + F_{\text{exp}}(\vec{u}) \]
  \[ F_{\text{poly}}(\vec{u}) = -\frac{1}{3!} \tilde{\kappa}_{abc} u^a u^b u^c + \frac{1}{2} a_{ab} u^a u^b + b_a u^b + \frac{\zeta(3) \chi(\tilde{X})}{2(2\pi i)^3}, \quad \tilde{X}: \text{mirror of } X \]

• To compute the periods we need to identify an integral basis of 3-cycles.
• Coefficients of \( F_{\text{poly}}(\vec{u}) \) depend on the geometric data of holomorphic cycles in \( \tilde{X} \).

Mirror Symmetry:

\[ \begin{array}{c|c}
\text{Integral 3-cycles in } X & \text{Integral 2n-cycles in } \tilde{X} \\
\end{array} \]
Periods of 3-cycles

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\]

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\mathcal{F}_{\text{exp}}(\vec{u}) = -\frac{1}{(2\pi i)^3} \sum_{C \in \mathcal{M}(\tilde{X})} n_C^0 \text{Li}_3(\text{e}^{2\pi i \vec{C} \cdot \vec{u}})
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where \(n_C^0\) are the genus zero Gopakumar-Vafa invariants. 

[Gopakumar, Vafa, hep-th/9809187]  
[Gopakumar, Vafa, hep-th/9812127]
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\[ \mathcal{F}_{\text{exp}}(\vec{u}) = -\frac{1}{(2\pi i)^3} \sum_{C \in \mathcal{M}(\tilde{X})} n_0^C \text{Li}_3(e^{2\pi i\vec{C} \cdot \vec{u}}) \]

where \( n_0^C \) are the genus zero Gopakumar-Vafa invariants.

- Existing methods allow for computing \( n_0^C \) when \( h^{2,1} \leq 5 \).
  - Also, hardly any results unless \( \mathcal{M}(\tilde{X}) \) is smooth and simplicial.

[Greene, Plesser, '90]
[Candelas, De La Ossa, Green, Parkes, '90]
[Batyrev, alg-geom/9310003]
[Hosono, Klemm, Theisen, Yau, hep-th/9308122]
[Hosono, Klemm, Theisen, Yau, hep-th/9406055]
... and more
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• Existing methods allow for computing \( n_0^C \) when \( h^{2,1} \lesssim 5 \).
  • Also, hardly any results unless \( \mathcal{M}(\tilde{X}) \) is smooth and simplicial.

• **Now: can compute** \( n_0^C \) **systematically for** \( h^{2,1} = \mathcal{O}(10) \).
  • No requirements on \( \mathcal{M}(\tilde{X}) \),
  • Up to \( h^{2,1} = \mathcal{O}(100) \) for some curves!

[Gopakumar, Vafa, hep-th/9809187]
[Gopakumar, Vafa, hep-th/9812127]
[MD, Kim, McAllister, Moritz, Rios Tascon, work in progress]
Is a de-Sitter solution possible in quantum gravity?
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**Ultimate Goal:** An *explicit* construction of a dS vacuum.

Is a de-Sitter solution possible in quantum gravity?

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Requires:

1. Exponentially small flux superpotential $W_0$

2. Strongly warped throat

3. Non-perturbative effects to stabilize Kähler moduli

4. Anti-D3 brane to uplift
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• A leading candidate: the KKLT scenario. [Kachru, Kallosh, Linde, Trivedi, hep-th/0301240]

Requires:

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   • Until recently: $\langle \bar{W}_0 \rangle \sim 10^{-2}$ [Denef, Douglas, Florea, hep-th/0404257]
   [Denef, Douglas, Florea, Grassi, Kachru, hep-th/0503124]
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2. Strongly warped throat
   - Can analytically continue to near a conifold point and use the same algorithm.
   - Bonus: a method for obtaining orientifolds at $h^{1,1} = \mathcal{O}(100)$. \[\text{[MD, Kim, McAllister, Moritz, hep-th/2009.03312]}\]
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THANK YOU!