NEW RESOLUTIONS FOR F-THEORY & CHIRAL MATTER IN STANDARD MODEL-LIKE CONSTRUCTIONS

upcoming work w/:
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INTRODUCTION

- F-theory on elliptic CY4 is a powerful toolkit for constructing SM-like vacua in string theory.

- Landscape questions $\Rightarrow$ tools to study large families of vacua.

- 4D: $G_4$ flux needed for chiral matter.

- New procedure (synthesis of known techniques) to compute fluxes for large class of hypersurface CY 4-folds (non-toric bases, non-Weierstrass).

- \((SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6\) models

[Beasley-Heckman-Vafa 0802.3391]
[Donagi-Wijnholt 0802.2969]
[Lin-Weigand 1406.6071, 1604.04742]
[Grassi-Halverson-Shaneson-Taylor 1409.8285]
[Cvetkovic-Halverson-Lin-Tian 1903.00009]
[Taylor-Turner 1906.11092]
[others]
PLAN FOR TALK

Ⅰ. Tuned gauge symmetry, generic matter, and \((SU(3) \times SU(2) \times U(1)) / \mathbb{Z}_6\)

Ⅱ. Resolving the \((SU(3) \times SU(2) \times U(1)) / \mathbb{Z}_6\) model

Ⅲ. Flux backgrounds for resolved elliptic CY 4-folds

Ⅳ. Selected results and future prospects
I. Tuned gauge symmetries, generic matter and $(SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$

- Singular elliptic CY 4-fold
  \[ X = \{ \sqrt[3]{2y^2 - (x^3 + f\, x^2 + g\, z^3)} = 0 \} \]

- "Tune" $(f, g, \Delta) : X$ with gauge symmetry $G$, Kodaira singularity (local) universal feature.

- When is matter $R$ "generic"?

- (F-theory / 6D (1,0) SUGRA definition): For fixed $G$,
  \# of tensors, $R$ is on branch of moduli space w/ highest dimension.

*assuming small anomaly coeffs.*
Tuned $\left( SU(3) \times SU(2) \times U(1) \right) / \mathbb{Z}_6$ model

- MSSM generic for SM group w/ discrete quotient
  \[ SU(2) \quad \text{and} \quad SU(3) \]

- Characteristic data: $(K_B, \Sigma_2, \Sigma_3, Y)$

- Constructions: unHiggs $q = 4$ $U(1)$ model

- [Taylor- Turner 1906.11092]

- Field theory, Higgs $SU(4) \times SU(2) \times SU(2)$ on $(4, 1, 2), (1, 3, \bar{2})$

- $Y = 0$, $F_1$ toric hypersurface

- [Taylor- Turner- Raghuram 1912.10941]

- $R = R_{MSSM} \oplus R_{\text{exotic}} \oplus R_{\text{adjoint}}$

- $R_{MSSM} = (3, 2)_{1/6} \oplus (3, 1)_{-1/3} \oplus (3, 1)_{2/3} \oplus (1, 2)_{1/2} \oplus (1, 1)$

- $R_{\text{exotic}} = (3, 1)_{-4/3} \oplus (1, 2)_{3/2} \oplus (1, 1)_2$

- [Klevers- Peña- Dehlmann- Piragua- Reuter 1408.4808]
II. Resolution: Cubic in \( \mathbb{P}^2 \)

- \( q = 4 \) model, un Higgs \( a_1 = s_3 = 0 \)

\[
X = \left\{ u \left( s_1 u^2 + s_2 uv + s_3 v^2 + s_5 uw + s_6 vw + s_8 w^2 \right) \\
+ (a_1 v + b_1 w)(d_0 v^2 + d_1 vw + d_2 w^2) = 0 \right\}
\]

- Hypersurface in \( Y : \mathbb{P}(L_u \oplus L_v \oplus L_w) \rightarrow B \)
  
  \[ [L_u] = \Sigma_3 - Y, \quad [L_v] = -K_B + \Sigma_2 - Y, \quad [L_w] = -K_B + \Sigma_3 \]

- Resolution \( X_5 \xrightarrow{\varphi} B \), five blow-ups

- Rational (not holomorphic) zero, generating sections
  
  \( \hat{D}_0, \hat{D}_1, \quad \varphi^*(\hat{D}_1 - \hat{D}_0) = Y \)
III. Flux background for resolved CY4

- Local matter, $C_R = \{ \Delta_i = \Delta_j = 0 \}$

- Chiral index, $\chi(R) = \int_{C_R} G_4, \quad G_4 \in H^4(\mathcal{X})$

  [for review cf. Weigand 1806.01854]

- $H^{2,2}_{\text{ver}} = \text{span}(H^{1,1} \wedge H^{1,1}), \quad H^{1,1} = \text{span} \{ \hat{D}_I = A, \alpha, \iota \}$

- $\chi(R) = C_{\mathcal{R}}^{I J} \Theta_{I J} = C_{\mathcal{R}}^{I J} \hat{G} \cdot \hat{D}_I \cdot \hat{D}_J$

- Unbroken Poincaré × gauge $\Rightarrow \Theta_{\alpha \beta} = 0.$

  [Dasgupta-Rajesh-Sethi 9408088]
  [Grimm-Hayashi 1111.1232]
  [Cveti\'\v{c}-Grimm-Klevers 1210.6034]

\[ \Theta_{I J} = G^{K L} \bar{\Omega}_{I J K L} + \Theta_{I J} \]

\[ \bar{\Omega}_{I J K L} = \hat{D}_I \cdot \hat{D}_J \cdot \hat{D}_K \cdot \hat{D}_L - W_{0 I J} \cdot W_{K L} - W_{I J} \cdot W_{0 K L} + W_{0 I} \cdot W_{I J} \cdot W_{K L} - (W_{I J}, i) \cdot W_{K L} \]

$\bar{\Omega}_{I J K L}$ for $\tilde{X}$ elliptic w/ generating section.
Flux background & intersection theory

- $\hat{\Theta} \rightarrow \Theta$ computable via intersection numbers $\hat{D}_i \cdot \hat{D}_j \cdot \hat{D}_k \cdot \hat{D}_l$

- For resolution $\tilde{X} \rightarrow X$, $X$ hypersurface of $\mathbb{P}(\bigoplus L_i) \rightarrow B$, push forward $\Phi^* (\hat{D}_i \cdot \hat{D}_j \cdot \hat{D}_k \cdot \hat{D}_l)$ to Chow ring of $B$

- Theorem: Given $\mathbb{P}(\bigoplus L_i) \rightarrow B$, formal power series $Q(t) = \sum_a \pi^* Q_a t^a$, $Q(t) = \sum_a Q_a t^a$

$\pi^* \tilde{Q}(H) = \sum_{i=1}^{3} \frac{Q([L_i])}{\prod_{j \neq i} ([L_i] - [L_j])}$, $H = c_1(O_{P^2})$

[ PJ - Taylor - Turner, to appear ]

Similar theorem for exceptional divisors $Y_i \xrightarrow{\hat{E}_i} 0 Y_{i-1}$

[ Esole - PJ - Kang 1703.00905 ]
[ Aluffi 0809.2425 ]
3D Chern-Simons Terms & 4D Anomalies

- 1-loop CS terms in 3D KK theory:
  \( \Omega_{i j}^{3D} = \sum_k b_{i j}(R) \chi(R) = \Omega_{i j}^{M\text{-theory}} \)

- Anomalies (GS mechanism):
  \( A^{i j}_a \partial_{i j} = 0 \)
  \( \Rightarrow \quad \bar{M} \) defined by \( \varepsilon_{i j} \bar{M} \varepsilon_{k l} = \partial_a \hat{\partial}_a \hat{A}_k \hat{A}_l \)
  \( \text{rank}(\bar{M}) \leftrightarrow \# \text{ anomaly solutions} \)

- Matching w/ f.t., get invertible linear system, can solve for
  \( \chi(R) = \chi_{i j}(R) \bar{\Omega}_{i j} \)

- Rational zero section \( \hat{D}_0 \Rightarrow M_{KK} \leq M_{W, \text{ Coulomb}} \)
  so contribution from KK modes needed for \( \Omega_{i j}^{3D} \)
**IV. Selected results**

- Example: $SU(5)$ Tate model, $R = 10 + 5$

\[
\chi(5) = \chi(10) = \frac{\alpha}{5} K_B \cdot (6 K_B + 5 \Sigma) \cdot 3
\]

for any smooth $B$, $SU(5)$ divisor $\Sigma \subset B$

- $(SU(3) \times SU(2) \times U(1)) / \mathbb{Z}_6$

MSSM + exotic matter not in Swampland

No obvious constraints on chirality, pending further analysis.

$Y = 0$ results agree with F11 model results
Future prospects

- Other SM-like constructions (flux-broken $G$?)
- Superpotentials
- More general flux backgrounds ($G_4 \in H_{rem}, H_{hor},$ flat $C_3$ configurations)
- More general resolutions, arbitrary dimension

THANK YOU!